TRANSIENT ANALYSIS OF MICROWAVE ANTENNAS BY METHOD OF MOMENTS: FEEDING PORTS AND EFFICIENCY OF MOT AND MOO METHODS

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ABSTRACT

The paper is focused on the time domain modeling of antennas by the method of moments. For this modeling, a scheme with weighted Laguerre polynomials is used. For efficient using of this scheme, the scaling factor of the time axis and the number of temporal basis functions has to be determined properly. Otherwise, the scheme is inaccurate and time-consuming.

In this paper, several clues how those quantities could be chosen are proposed. Further, there is a discussion how to model feeding ports of symmetrical antennas to get a shorter transient response and so an analysis is less time consuming. Using of proposed approaches is shown on an symmetric strip dipole.

1. INTRODUCTION

Electric field integral equation (EFIE) is widely used for the numerical analysis of electromagnetic radiation and scattering. Essentially, when broad-band information is desired, it is more efficient to solve the electromagnetic problem in the time domain. Several formulations have been presented, the marching-on in time method (MOT) - the explicit and implicit schemes [1], and the marching-on in order method (MOO) - the scheme with Laguerre polynomials [2], [3], [4].

The scheme with Laguerre polynomials uses weighted Laguearre polynomials as basis functions for the temporal approximation of unknown current density. There are five characteristic properties of the weighted Laguerre polynomials: they are (1) causal, (2) recursively computed, (3) orthogonal, (4) convergent, and (5) separating space and time variables. Thanks to their properties, unconditionally stable scheme is derived. The own idea of this procedure is that a temporal response is approximated by a set of weighted Laguarre polynomials.

2. SCHEME WITH LAGUERRE POLYNOMIALS

Let S denote the surface of a closed or open perfectly electric conducting (PEC) body illuminated by a transient electromagnetic wave. This incident wave induces a surface current $\mathbf{J}(\mathbf{r},t)$ on S. The scattered electric field $\mathbf{E}^{\mathrm{S}}(\mathbf{r},t)$ computed from the surface current is given by

$$\begin{aligned}&\frac{\mu}{4\pi}\int_S \frac{\partial \mathbf{J}\left(\mathbf{r}',t-\dfrac{R}{c}\right)}{\partial t}\frac{1}{R}dS' \\ &-\frac{\nabla}{4\pi\varepsilon}\int_S \frac{\int \mathbf{J}\left(\mathbf{r}',t-\dfrac{R}{c}\right)dt}{R}dS' = -\mathbf{E}^{\mathrm{S}}(\mathbf{r},t)\end{aligned} \quad . \tag{1}$$

The permittivity and permeability of the surrounding medium are μ and ε, respectively, $R=|\mathbf{r}-\mathbf{r}'|$ is the distance between an arbitrarily located observation point $\mathbf{r}$ and source point $\mathbf{r}'$ on S, and $\tau = t-R/c$ is the retarded time. The velocity of propagation in the surrounding medium is $c=(\mu\varepsilon)^{-1/2}$. Since the total tangential electric field is zero on the conducting surface for all times, we have

$$\left[\mathbf{E}^{\mathrm{i}} + \mathbf{E}^{\mathrm{S}}(\mathbf{J})\right]_{\tan} = 0, \mathbf{r} \in S\,, \tag{2}$$

where $\mathbf{E}^{\mathrm{i}}$ is the incident electric field on the scatterer and the subscript "tan" denotes a tangential component.

The equation (2) is solved by the method of moments (MoM) [1]. The analyzed structure is approximated by planar triangular patches and the RWG functions [5] are used to expand the spatial variation of the electric current density. The unknown current density $\mathbf{J}(\mathbf{r}, t)$ is expanded with unknown current coefficient $I_{\mathrm{n,u}}$, weighted Laguarre polynomial φ_{u} (of the order u) and spacious basis function $\mathbf{f}_{\mathrm{n}}$

$$\mathbf{J}(\mathbf{r},t) = \sum_{\mathrm{n}=1}^{\mathrm{N}}\sum_{\mathrm{u}=0}^{\infty} I_{\mathrm{n,u}}\varphi_{\mathrm{u}}(ts)\mathbf{f}_{\mathrm{n}}(\mathbf{r})\,, \tag{3a}$$

$$I_n(t) = \sum_{u=0}^{\infty} I_{n,u}\varphi_u(ts)\,, \tag{3b}$$

where N is the number of nonboundary edges [5] and s is a time scale factor. By substituting (3) to (2) and after testing procedure in the time and space, the unknown current coefficients can be computed according to the following matrix equation

$$\left[\alpha_{\mathrm{mn}}\right]\left[I_{\mathrm{n,v}}\right] = \left[V_{\mathrm{m}}^{\mathrm{v}}\right] - \left[\beta_{\mathrm{m}}^{\mathrm{v}-1}\right],\ v = 0,1,2...N_{\mathrm{L}}\,, \tag{4}$$

where

$$V_{\mathrm{m}}^{\mathrm{v}} = \int_0^{\infty} V_{\mathrm{m}}(t)\varphi_{\mathrm{v}}(st)s\,dt\,, \tag{5}$$

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and $[\alpha_{mn}]$ denotes a matrix whose elements don't depend on the order of the testing function $\varphi_v(st)$, $[\beta_m^{v-1}]$ is the matrix which contains computed terms of order from 0 to v-1 and N_L is the number of temporal basis functions The upper limit of infinity can be replaced by a finite time interval T_t. This interval is chosen in such a way that the waveforms of interest $V_m(t)$ have practically decay to zero at that time. The matrix equation (4) is solved N_L times. Detailed steps of the derivation are written in [2], [3] and [4].

3. CHOICE OF SCALING FACTOR AND NUMBER OF TEMPORAL BASIS FUNCTIONS

For efficient using of the scheme (4), the scaling factor s and the number of temporal basis functions N_L have to be determined properly. Otherwise, the scheme is inaccurate and time-consuming. However, both these quantities mainly depend on the excited signal and the properties of an analyzed structure.

By our experience for a Gaussian pulse (7), the time scale factor can be evaluated by

$$s \cong 7B\,, \tag{6}$$

where B is the frequency at which the magnitude of the spectra of Gaussian pulse descends to 3% of the magnitude at frequency of 0 Hz. Gaussian pulse is defined (see below)

$$U(t) = U_0 \frac{4}{\sqrt{\pi}T} e^{-\left[\frac{4}{T}(ct-ct_0)\right]^2}\,, \tag{7}$$

where T is the width of Gaussian pulse in LM (1 LM is the length of time that it takes for the electromagnetic wave to travel 1 m in a medium with the velocity of light c).

In [2] it is recommended to determine the minimum number of temporal basis functions as follows

$$N_L = 2BT_f + 1\,, \tag{8}$$

where B has a similar meaning as in (6) and T_f is the length of response. However, when the unknown structure is analyzed, the length of response is unknown. The equation (8) is almost inapplicable.

So when we use the scheme (4), we do not determine N_L before the using of the algorithm but when it is running by checking the value of responses at the beginning and the end of response (we look for the end of response) in the time domain. For more accurate results, it is possible to check the current or the input impedance in the frequency domain up to the high frequency of our interest. According to our experience, it is enough when the current is checked at the lowest and highest frequencies of our excited signal.

Several clues how or where to check responses in the time or frequency domain will be shown on an example of the symmetric strip dipole (Figure 1) excited at its centre by Gaussian pulse (the delta-function generator) with the following parameters: U_0=120π V, T=0,2 LM a ct_0=0,3 LM. The body of dipole is modeled by 40 triangular patches. B=3.6 GHz, s=2.52e+10. For comparison of accuracy, the same problem was solved by the method of moments in the frequency domain [5], [6], [7] because the accuracy of this approach is known and after that the data were inversely Fourier transformed.

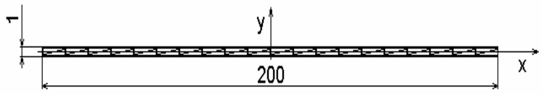


Figure 1. The symmetric strip dipole

3.1 TIME DOMAIN

In this part, a clue is shown how to find the end of response which sufficiently accurately approximated by a certain number of polynomials. For this example, we choose N_L=241. The temporal response is depicted in the Figure 2a) and the enlarged details, the beginning and the end of the response are depicted in the Figure 2b) and 2c).

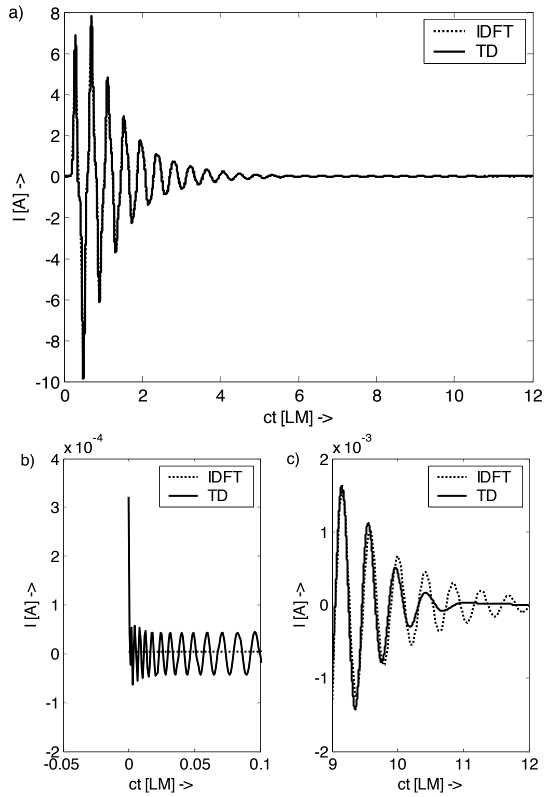


Figure 2. The temporal response at the centre of the symmetric strip dipole(N_L=241). a) The whole response, b) the enlarged detail at the beginning of the response, c) the enlarged detail at the end of the response.

At the beginning and the end of response, there are the biggest inaccuracies. They can be reduced by increasing of number polynomials, however, this is time consuming. So it is appropriate to determine an error before the computing which is acceptable. This error can be set e.g. as the ratio of the values at the beginning and the end of the response to the highest value of response.
At the beginning of the response, the value can be checked at *ct*=0 LM, because there is the highest inaccuracy. However, it is not so easy for the end of response, because the end is not known. For this reason, it can be helpful the N_L-th weighted Laguerre polynomial (Figure 3) because according to our experience, the temporal response is sufficiently approximated by N_L polynomial, up to the time where the N_L-th weighted Laguerre polynomial has the 8th local extreme (computed from the end *ct*=12 LM), for this case *ct*=9.6 LM.

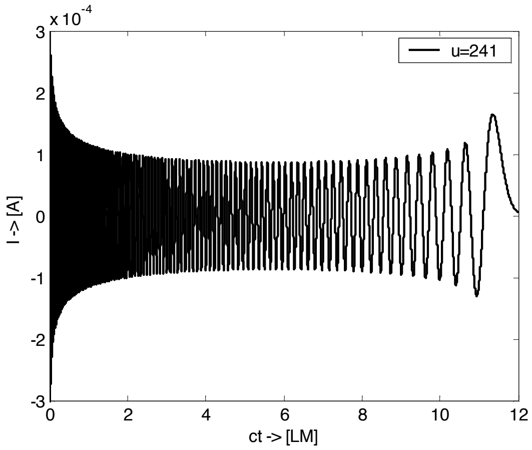
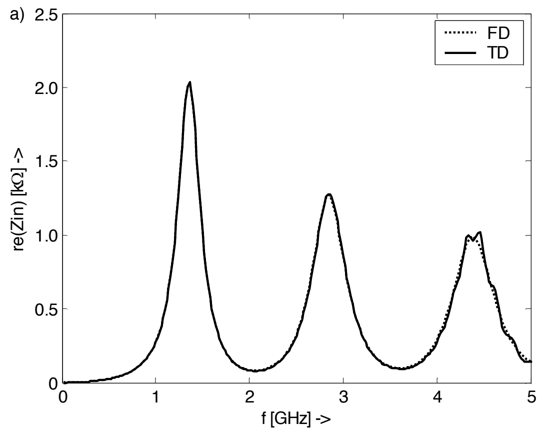


Figure 3. N_L-th weighted Laguerre polynomial multiplied by corresponded unknown current coefficient $I_{n,u}$

3.2 FREQUENCY DOMAIN

The input impedance computed from the response in the Figure 2a) is depicted in the following Figure 4. In comparisons with the solution obtained by solving the same structure in the frequency domain it is seen, that both solution fits very well up to 3.5 GHz.
There is an issue. How to recognize that the input impedance or the current at the certain frequency is accurate enough? Possible answer can be got by the Figure 5.
In the Figure 5a) the development of the real part of the current at the frequency 3 GHz on the number of temporal basis functions is depicted. (This current was computed according to (3) after its Fourier transformation and multiplication by the width of the dipole.) The computed value by the scheme (4) are depicted by the dotted line, and by the solid line these values are fitted in the interval of *N* points (for this case *N*=8) by the first order polynomial.



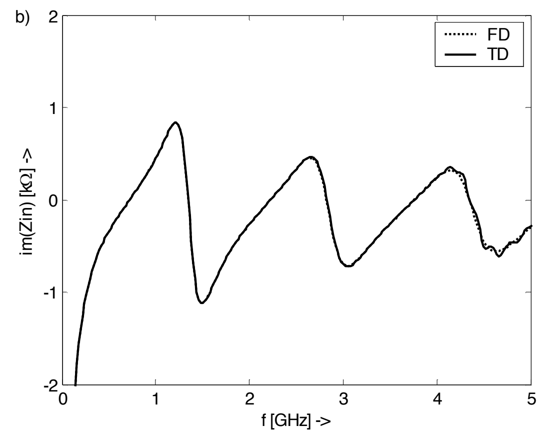


Figure 4. The input impedance of the strip dipole (N_L=241). a) The Real part of the input impedance, b) the imaginary part of the input impedance.

For fitting the least square method is used. For the first order polynomial it can be written

$$\mathrm{re}(I) = a_1(u - N(M-1)) + a_0\,, \tag{9}$$

where *M* is the *M*-th interval.
In the Figure 5a) it is seen that the real part of the current gradually go towards the certain (correct) value. The development of the fitting coefficients a_1 (the slope of lines) is depicted in the Figure 5c). It seen that the value of these coefficients gradually go towards zero.
So these ones can be the clues how to determine the number of the polynomial N_L, when the algorithm is running or to say how preciously the current or the input impedance at a certain frequency is computed. So the N_L or the error can be determined from the values of slopes and the error of approximation. According to our experience, it is good if the *N* is chosen from 5 to 12. Of course, it has to be checked the imaginary part too.
Similar development of the current on the number of polynomial can be expected in the time domain at the certain time *t* or *ct*.

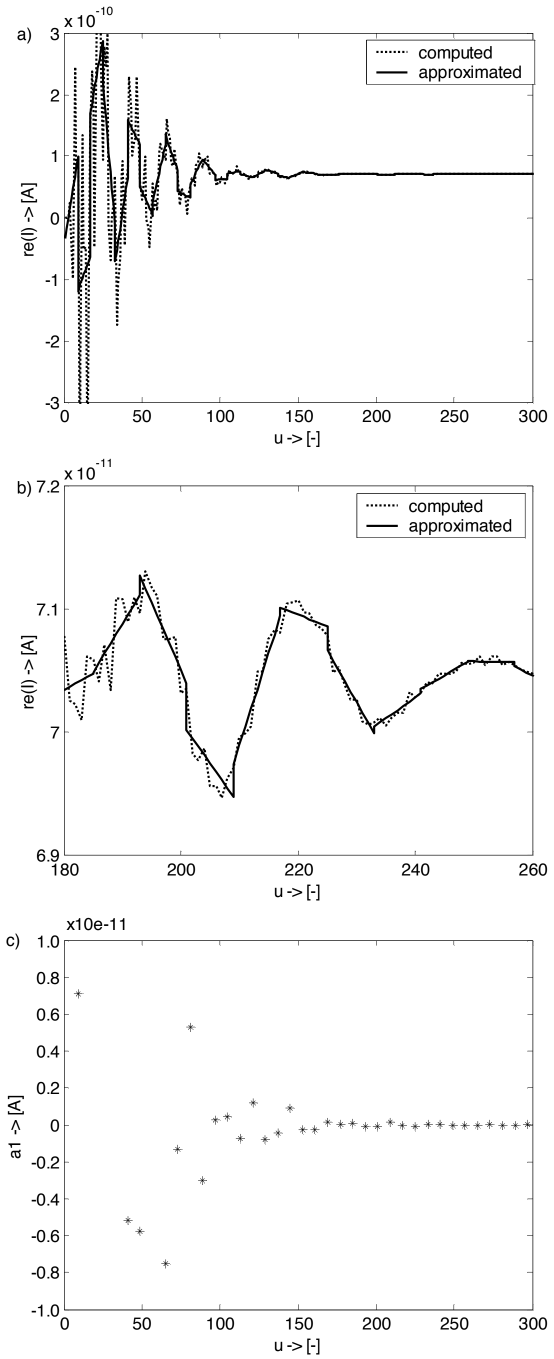


Figure 5. The development of the real part of the current at the frequency 3 GHz on the number of temporal basis functions. a) The whole development for u from 0 to 300, b) the enlarged detail of the development for u from 180 to 260, c) the slope (coefficient a_1) of approximated lines.

4. FEEDING PORTS

In the previous part, the symmetric strip dipole was fed at its centre by the delta-function generator (the feeding edge model) [7] depicted in the Figure 6a). This model assumes a gap of negligible width d. If the voltage across gap is U, then the electric field within the gap becomes

$$\mathbf{E} = \frac{U}{d}\mathbf{n}_{\mathrm{y}}, \tag{10}$$

When d tends to zero, (10) predicts infinite values within the gap - the delta-function approximation

$$\mathbf{E} = U\delta(y)\mathbf{n}_{\mathrm{y}}, \tag{11}$$

It is convenient to associate the gap with an inner edge h of he boundary element structure. There is only one RWG element corresponding to that edge. Thus the "incident" electric field will be zero everywhere expect for one RWG element h. So after a spacious testing procedure, it can be written

$$\begin{array}{ll} V_{\mathbf{m=h}} = l_{\mathbf{h}}U & \text{for edge element } m = h \\ V_{\mathrm{m=h}} = 0 & \text{otherwise} \end{array}, \tag{12}$$

where l_{h} is the length of the l-th edge.

When the delta-function generator was used in the previous example, the transient response (Figure 2a)) was quite long. The reason of this is that most energy given by the source, after the feeding by the pulse, do not radiate from the dipole, but is only reflected from one end of the arm of dipole to the other and back. So it would be fine, if the energy could be "sucked out" if it isn't radiated.

A possible answer how to realize the "sucking" port is depicted in the Figure 6b). Each arm of dipole (the distance of the arms is b) is excited by the delta-function generator (with respect the sign) and both these arms are matched by a terminating line.

The idea of numerical imposing matching termination is simple: the current I(t) propagating along a lossless transmission line can be expressed as a sum of a forward traveling current wave and a backward one. On a match-terminated line, there are no reflections from the load. So for matching the termination line (Figure 6b)) to the reference plane the following conditions can be written

$$I_{\mathrm{hk}}(t) = I_{\mathrm{hk-1}}\left(t - \frac{\Delta x}{c}\right), \text{for } k = 2, 3, 4\,, \tag{13a}$$

$$I_{\mathrm{lk}}(t) = I_{\mathrm{lk-1}}\left(t - \frac{\Delta x}{c}\right), \text{for } k = 2, 3, 4\,. \tag{13b}$$

The relations (13) have to be added to N equations (4). These conditions can be simplified with the consideration

$$I_{\mathrm{hk}}(t) = -I_{\mathrm{lk}}(t)\,. \tag{14}$$

Details about matching conditions in the time domain are written in [4].

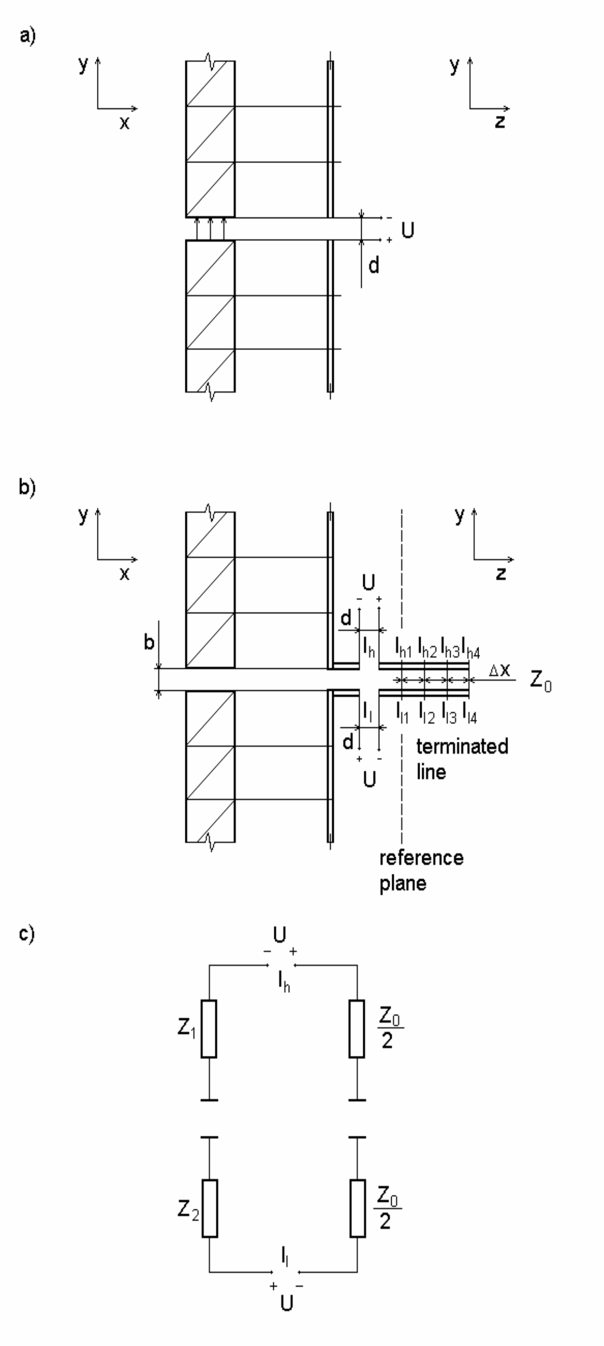


Figure 6. The models of feeding ports. a) The delta-function generator, b) the delta-function generators with terminated line c) the equivalent circuit of the delta-function generators with terminated line.

The input impedance can be computed with the consideration the Figure 6c)

$$Z_{\text{in}} = Z_1 + Z_2 = \frac{U}{I_\text{h}} + \frac{U}{I_\text{l}} - Z_0\,, \tag{15}$$

where Z_0 is the characteristic impedance of the terminated line. This one can be determined:

- analytically.
- numerically: A transmission line guiding a forward traveling wave is analyzed by the same method like an analyzed structure. If a transmission line is matched at both ends, only a forward traveling wave exists and the characteristic impedance can be computed.

The first approach is faster and usually less accurate than the second one. We prefer to use the second approach.

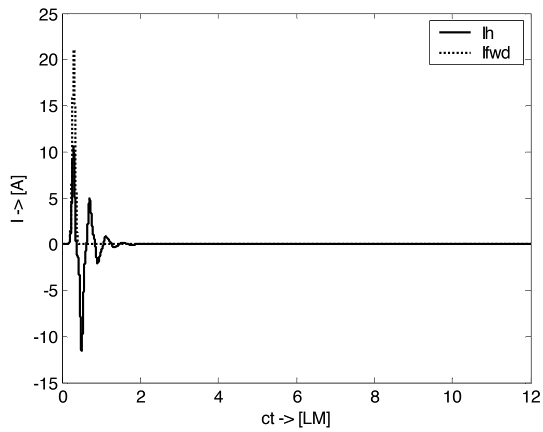


Figure 7. The current responses for the symmetric strip dipole (Figure 1) and the transmission line with the characteristic impedance Z_0. The feeding port depicted in the Figure 6b) was used. N_L=105.

Using of the proposal feeding model will be shown on the same structure as in the part 3. The transient responses are depicted in the Figure 7, I_h is the current defined in the Figure 6b) and I_fwr is the forward current wave on the matching line. It is seen, that the current response I_h is approximately 4 times shorter than the response in the Figure 2a).

The input impedance is depicted in the Figure 8, *FD a)* denotes the input impedance computed by the method of moments in the frequency domain and the feeding ports depicted in the Figure 6a) was used, and *TD b)* denotes the input impedance computed by the scheme (4), N_L=105, and the feeding ports depicted in the Figure 6b) was used. It is seen, that there is a frequency shift about 1%. This shift is due to the gap b=1 mm between arms of the dipole (Figure 6b)).

The number of temporal basis functions was determined during the run of the algorithm N_L=105 with the percentage error 3% from the frequency 0.32 GHz to 3.5 GHz. So using of the proposal port is much less time consuming then the classical delta-function generator.

To be noticed, the values of the real part of the input impedance of this dipole are units of Ohms at low frequencies, so the absolute error of tenths of Ohms makes a big percentage error.

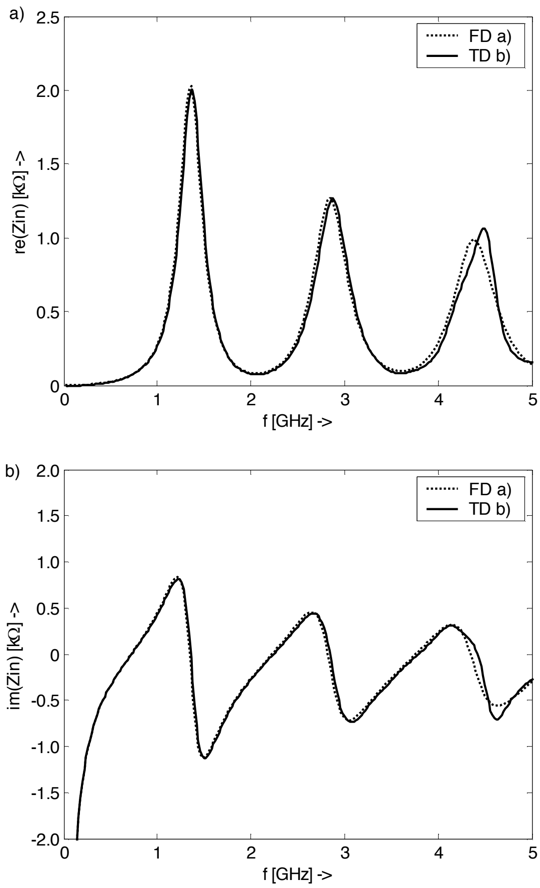


Figure 8. The input impedance obtaining by using the ports depicted in the Figure 6). a)The real part of the input impedance, b) the imaginary part of the input impedance.

5. EFFICIENCY OF MOT AND MOO METHODS

In this part there is a discussion about the efficiency of the implicit scheme (MOT - Marching On in Time method) and the scheme with the Laguerre polynomial (MOO -Marching On in Order method).

For both these schemes, an inverse matrix has to be once solved. Meanwhile, the inverse matrix at the implicit scheme is spare, but at the other scheme it is full. The computational complexity for the classical algorithm of the implicit scheme is N_tN^2, where N_t is the number of time steps, so it grows linearly with the length of response. However, the computational complexity for scheme with the Laguerre polynomial is $(N_LN)^2/2$, so it grows squarely with the number of temporal basis functions and the length of response. The advantage of the scheme with Laguarre polynomial is the fact that the space and the time variable can be separated, so one doesn't have to worry about the Courant condition. In addition, the derivative resp. integration with respect to time can be done analytically.

According to our experience, the scheme with Laguarre polynomial is usually more time consuming than the implicit scheme, however, it is accurate [4] and it doesn't suffer from late time oscillation.

6. CONCLUSION

In this paper, we described several clues how to choose the scale factor and the number of temporal basis for the efficient using a scheme with weighted Laguerre polynomials for modeling antennas in the time domain. Furthermore, we proposed how an feeding port for symmetrical antenna could be modeled in the time domain. By using of the proposed port, the transient response can be much shorter than by using the delta-function generator. In case of the symmetric dipole, the transient response was 4 times shorter.

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